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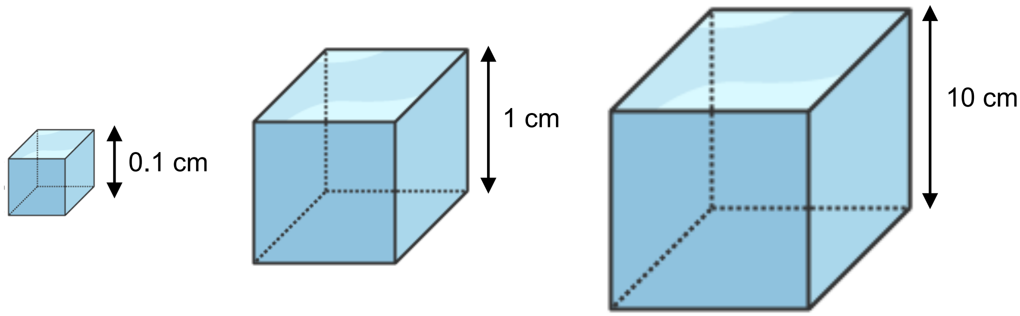
Math Activity for "Big and small animals with sticky feet" Data Nugget
Scaling up: How surface area and volume change with size

Let's consider how surface area and mass differ for large and small objects of the same shape. Imagine three cubes, each filled with water. The first cube measures 0.1 centimeter on a side, the second is 1 centimeter on a side, and the third is 10 centimeter on a side. This means that each cube is 10 times larger than the previous.



Below, calculate the **surface area** of one side (length x width), the **volume** (length x width x height), and **mass** (volume x density) of each cube. The density of water (H_2O) is 1.0 grams/cubic centimeter.

0.1 cm cube
Surface area of one side = 0.1 cm x 0.1 cm = **0.01 cm^2**
Volume = 0.1 cm x 0.1 cm x 0.1 cm = **0.001 cm^3**
Mass = 0.001 cm^3 x 1 g/cm^3= **0.001 g**

1 cm cube
Surface area of one side = 1.0 cm x 1.0 cm = **1.0 cm^2**
Volume = 1.0 cm x 1.0 cm x 1.0 cm = **1.0 cm^3**
Mass = 1.0 cm^3 x 1.0 g/cm^3 = **1.0 g**

10 cm cube
Surface area of one side = 10 cm x 10 cm = **100 cm^2**
Volume = 10 cm x 10 cm x 10 cm = **1000 cm^3**
Mass = 1000 cm^3 x 1.0 g/cm^3 = **1000 g**

From these calculations we see that while the size of each cube is 10 times larger than the previous cube, all the values do not increase in this way. The surface area of the side of each cube is 100 times larger than the side of the prior cube, while mass and volume are 1000 times larger than the mass and volume of the previous cube!

This pattern also holds for animals that are the same shape but different sizes. Their mass and volumes differ in the same non-proportional way compared to their surface area. As an example, consider three gecko lizard species that have the same body proportions, but each is 10 times longer than the previous. Geckos, a type of lizard, have sticky toes that allow them to climb almost anywhere including windows and tree branches. If the surface area of the toe pad is too small, the gecko will not be able to hold its body weight and will be unable to climb. We can use our previous calculations to understand how the surface area of gecko toe pads change in proportion to the mass of its body. Let's say our smallest gecko species has toe pads 0.0003 square millimeters in total area and a mass of 0.00002 grams.

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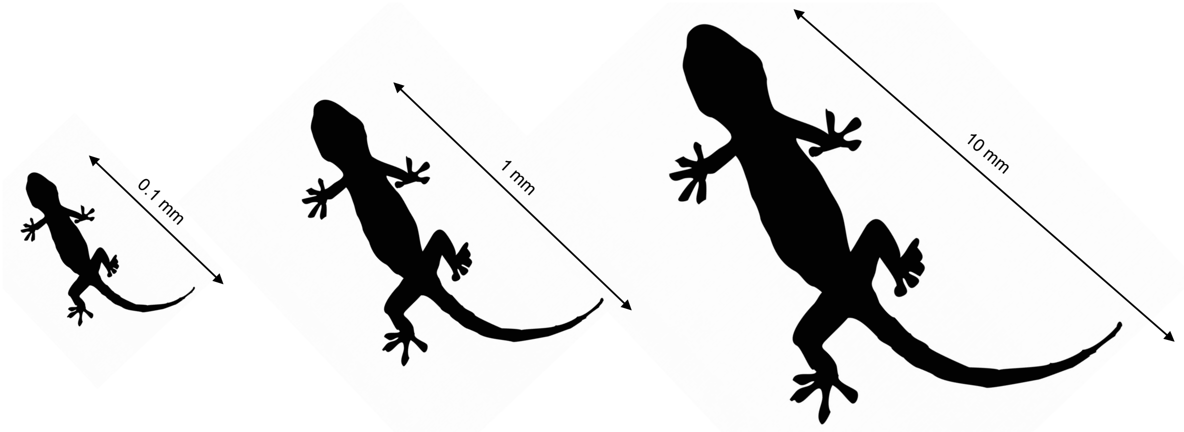

Small gecko species
Surface area of toe pads = 0.0003 mm^2
Mass = 0.00002 g

From the figure above we see that the medium sized gecko species is 10 times longer than the smallest gecko species. Based on our prior calculations, we know the medium gecko will have a total toe pad surface area that is 100 times larger than the smallest gecko species and a mass that is 1000 times larger than the smallest gecko species.

Medium gecko species
Surface area of toe pads = 0.0003 mm^2 x 100 = **0.03 mm^2**
Mass = 0.00002 g x 1000 = **0.02 g**

Using the figure and calculations from above, we see that the large gecko species is 10 times longer than the medium gecko species and will have toe pads with a total surface area that is 100 times larger than the medium gecko species and a mass that is 1000 times larger than the medium gecko species.

Large gecko species
Surface area of toe pads = 0.03 mm^2 x 100 = **3 mm^2**
Mass = 0.02 g x 1000 = **20 g**

To help understand the differences between species, we can calculate the **mass to toe pad surface area ratio** for each species. In our example, the smallest gecko species has to support its mass of 0.00002 grams using toe pads with an area of 0.0003 square millimeters. This means our small gecko species has a mass to area ratio of 0.067 g/mm^2 (0.00002 g / 0.0003 mm^2). This should be more than enough surface area for the little lizard to stay attached to surfaces like glass and tree branches. However, the medium species has a ratio of 0.67 g/mm^2 and the large species has a ratio of 6.7 g/mm^2. We can see that the large species has to support more mass with relatively smaller toe pads (g/mm^2 increases)! What does this mean for larger species?

Larger animals may have to compensate for their heavier mass by having extremely large toe pads. Natural selection would favor individuals with toe pads large enough to hold their weight, which may lead to the evolution of non-proportionately larger toe pads in larger species. By evolving non-proportionately larger toe pads, big species may maintain the same mass to area ratio as smaller species.